Spin and Chirality Orderings of Frustrated Magnets — Stacked-Triangular Antiferromagnets and Spin Glasses

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ABSTRACT

"Chirality" is a multispin quantity representing the sense or the handedness of the noncollinear spin structures induced by spin frustration. Recent studies have revealed that the chirality often plays an important role in the ordering of certain frustrated magnets. Here I take up two such examples, stacked-triangular antiferromagnets and spin glasses, where the inherent chiral degree of freedom affects underlying physics and might lead to novel ordering phenomena. The first topic is the criticality of the magnetic phase transition of vector (*i.e.*, XY or Heisenberg) antiferromagnets on the three-dimensional stacked-triangular lattice. The second topic is the nature of the spin-glass ordering. I will review the recent theoretical and experimental works on these topics, with particular emphasis on the important role played by the chirality.

§1. Introduction

Frustration often gives rise to new interesting phenomena in the magnetic ordering of spin systems. One interesting consequence of spin frustration in vector spin systems might be the possible appearance of "chiral" degrees of freedom. "Chirality" is a multispin quantity representing the sense or the handedness of the noncollinear spin structures induced by spin frustration. While the chirality has long been a familiar concept in the field of molecular chemistry, it was introduced into the field of magnetism first by Villain [1]. Recent studies have revealed that the chirality often plays a very important role in the magnetic ordering of frustrated spin systems.

In this article, I will take up two such examples, stacked-triangular antiferromagnets and spin glasses, where the inherent chiral degree of freedom leads to new interesting phenomena not encountered in unfrustrated magnets. The first topic is the criticality of the magnetic phase transition of vector (*i.e.*, XY and Heisenberg) antiferromagnets on the three-dimensional (3D) stacked-triangular lattice. Some time ago, the present author proposed that these frustrated triangular magnets, *e.g.*, $CsMnBr_3$ and VBr_2, might exhibit novel critical behavior, possibly lying in a new "chiral" universality class, distinct from the standard Wilson-Fisher universality class [2, 3, 4, 5]. While this prediction have been supported by most of subsequent experiments, other theorists indicated that the transition was weakly first order, and the subject still remains controversial [6]. In this article, I wish to review the present status of the study, and discuss what to be done in the future to further clarify the situation.

The second topic is the nature of the spin-glass ordering. Our present understanding of the nature of the experimentally observed spin-glass (SG) transition and of the SG ordered state still remains unsatisfactory [7]. Although the chirality has rarely been invoked in the standard scenario of the SG ordering, I with to explain here one promising scenario presented some time ago based on the spin-chirality decoupling-recoupling mechanism [8]. In this scenario, *chiral-glass order*, which is expected to occur in a fully isotropic 3D Heisenberg SG, plays an essential role. I examine some

of the consequences of this scenario, particularly in light of the experimental results on canonical SG.

§2. Chirality

Frustration in vector spin systems often gives rise to the noncollinear or noncoplanar spin orderings. Such canted spin structures generally give rise to the nontrivial chiral degrees of freedom. Chirality basically represents the handedness of such noncollinear (noncoplanar) spin structures. Two different types of chiralities have often been discussed in the literature: One is called a vector chirality and the other a scalar chirality.

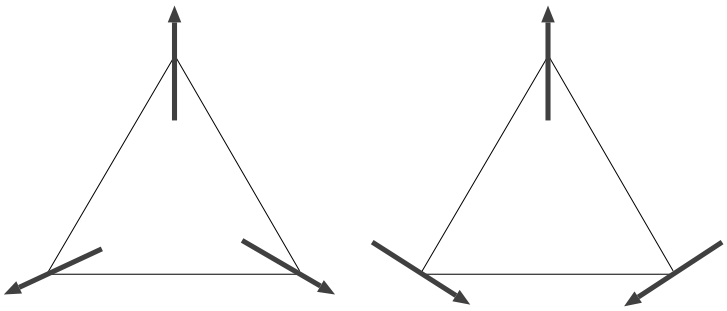

Figure 1: Two chiral states in the ground-state spin configurations of antiferromagnetically-coupled three vector spins on a triangle. These chiral states are characterized by the vector chirality.

Chiral states representing the right- and left-handed configurations are illustrated in Fig.1 for an example of three antiferromagnetically coupled XY spins located at each corner of a triangle. The ground-state spin configuration is a well-known 120° spin structure, in which each XY spin on a plane makes an angle equal to $\pm 120°$ with the neighboring spins. One may define the chirality of the first type, the vector chirality, via a vector product of the two neighboring spins, averaged over three spin pairs, by

$$\kappa = \frac{2}{3\sqrt{3}} \sum_{<ij>} \left[\vec{S}_i \times \vec{S}_j \right]_z .$$

Evidently, the sign of κ represents each of the two chiral states, *i.e.*, either a right-handed (clockwise) state for $\kappa > 0$ or a left-handed (counterclockwise) state for $\kappa < 0$. In the case of XY spins considered here, the chirality κ is actually a *pseudoscalar*: It remains invariant under global $SO(2)$ proper spin rotations while it changes sign under global Z_2 spin reflections. Hence, in order to transform one chiral state to the other, one needs to make a global spin reflection. The chiral order is closely related to the spontaneous breaking of a discrete Z_2 spin-reflection symmetry.

When the spins in Fig.1 are Heisenberg spins with three components, the ground-state spin configuration is again the coplanar 120° spin structure, whereas in contrast to the XY case there no longer exists a discrete chiral degeneracy. This is because the apparently right-handed chiral state can now be transformed into the apparently left-handed chiral state via a continuous spin rotation of π making use of the third dimension of the Heisenberg spin space. In this case, one may define the vector chirality as an *axial vector* by

$$\vec{\kappa} = \frac{2}{3\sqrt{3}} \sum_{<ij>} \vec{S}_i \times \vec{S}_j.$$

In the case of three-component Heisenberg spins, there are occasions in which the ordered state spin configuration is *noncoplanar* rather than coplanar. Such a non-coplanar ordered state is actually realized in a Heisenberg SG. In fact, a noncoplanar state can sustain a discrete chiral degeneracy even in the case of Heisenberg spins, which is characterized by mutually opposite signs of the chirality of the second type, the scalar chirality. It is a pseudoscalar defined for three neighboring spins by,

$$\chi = \vec{S}_i \cdot \vec{S}_{i+\delta} \times \vec{S}_{i+\delta'}.$$

An illustrative example is given in Fig.2.

These chiral degrees of freedom inherent to the noncollinear or noncoplanar spin orderings often deeply affect the ordering of frustrated magnets, leading to novel phenomena not encountered in conventional unfrustrated magnets, as we shall see in the next two sections.

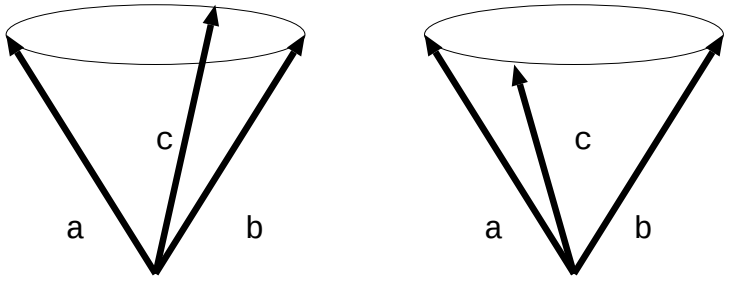

Figure 2: Two chiral states associated with the noncoplanar spin structure formed by three-component Heisenberg spins. These chiral states are characterized by the scalar chirality.

§3. Critical properties of stacked-triangular antiferromagnets

In this section, I wish to deal with the novel critical behavior of XY and Heisenberg antiferromagnets on the 3D stacked-triangular lattice. The stacked-triangular lattice consists of 2D triangular layers stacked in register along the orthogonal direction directly on top of each other. There are several experimental realizations of the stacked-triangular antiferromagnet with nontrivial chiral degree of freedom. Examples are ABX_3-type compounds $CsMnBr_3$ and $CsVBr_3$, or vanadium dihalides VCl_2 and VBr_2, *etc*. Even Ising-like ABX_3-type compounds with an easy-axis-type anisotropy, such as $CsNiCl_3$, $CsNiBr_3$ and $CsMnI_3$, exhibit the chiral critical behavior under external fields higher than a certain critical field.

The ordered-state spin configuration in these chiral stacked-triangular antiferromagnets is a three-sublattice 120° spin structure in each triangular layer. As was illustrated for the case of an isolated triangle, such an ordered state possesses a nontrivial chiral degree of freedom. This leads to the appearance of a new symmetry distinct from that of the standard unfrustrated magnets, and may give rise to magnetic phase transitions of new universality class. Indeed, the present author suggested the possible occurrence of such a new universality class, chiral universality class, on the basis of a symmetry argument [2, 4], Monte Carlo simulations [2, 5] and renormalization-group (RG) calculations including the $\epsilon = 4 - d$ and $1/n$ expansions

[3]. As usual, we mean here by n and d the number of spin components and the spatial dimensions, respectively, where the model is now extended to general n-vector spins on a d-dimensional stacked-triangular lattice in which the 2D triangular layers are stacked in the remaining $d-2$ directions in a hypercubic fashion. Theoretical analyses of the critical properties of n-vector stacked-triangular antiferromagnets performed by various authors, however, lead to often ambiguous, and sometimes mutually conflicting results. Below, I will refer to several points of special interest, leaving other details to the review article [6].

The RG $\epsilon = 4 - d$ expansion based on an appropriate Landau-Ginzburg-Wilson Hamiltonian yields a new fixed point associated with the noncollinear criticality, distinct from the standard Wilson-Fisher fixed point. This new fixed point, which we call a chiral fixed point, exists stably in a certain region of the parameter space, at least for sufficiently large n meeting the condition, $n > 21.8 - 23.4\epsilon + 7.1\epsilon^2 + O(\epsilon^3)$ [3, 9]. For n smaller than this critical value, the transition is likely to be first order. The $1/n$ expansion result is also consistent with the $\epsilon = 4 - d$ expansion result, yielding a continuous transition with a set of novel critical exponents for any $2 < d < 4$ [3]. The question still remaining is what happens at the physical point, *i.e.*, $n = 2$ or 3 at $d = 3$. Two possibilities seem to exist: (i) The critical value $n_c(d = 3)$ is smaller than, or equal to the physical value $n = 2$ or 3. If this is the case, a real system exhibits a continuous transition of new chiral universality. (ii) The critical value $n_c(3)$ is greater than the physical value $n = 2$ or 3. If this is the case, a real system exhibits a first-order transition.

The present author originally suggested, partly based on an extrapolation of the low-order ϵ-expansion result, but mainly based on the Monte Carlo observation, that the case (i) was a rather likely situation [3]. By contrast, some other theorists argued that the case (ii) was more likely: For example, on the basis of the Borel-Padé resummation of the ϵ-expansion, Antonenko *et al* estimated the critical value as $n_c(3) \simeq 3.39$ which turned out to be slightly above the physical value $n = 3$. Thus, the subject

still remains controversial.

Almost all Monte Carlo (MC) simulations so far made on the XY ($n = 2$) and Heisenberg ($n = 3$) stacked-triangular antiferromagnets yielded a continuous transition characterized by the set of novel exponents distinct from the standard $O(n)$ exponents [5, 10, 11, 12]. We show in Table I various critical exponents and specific-heat amplitude ratio as determined by several MC simulations for the stacked-triangular Heisenberg ($n = 3$) antiferromagnet. As is clear from Table I, the MC values reported by different authors agree with each other, but are inconsistent with the standard Heisenberg values. They also deviate considerably from the $O(4)$ or the mean-field tricritical values suggested in Ref.[13] on the bases of the RG $\epsilon = d - 2$ expansion.

Since the simulations are performed for finite lattices, one cannot completely rule out the possibility that an indication of a first-order transition eventually develops for still larger lattices. A signature of such a first-order transition, however, has not been observed up to the linear size $L = 60$. Thus, the MC results so far obtained are in favor of the chiral-universality scenario. We note in passing that the numerical situation in the XY ($n = 2$) case is also similar to the Heisenberg ($n = 3$) case: See Ref.[6] for details.

We have included in Table I the chirality exponents of the Heisenberg stacked-triangular antiferromagnet as determined by MC, where β_κ and γ_κ refer to the chiral long-range order and the conjugate chiral susceptibility exponents, respectively. The RG analysis of Ref.[3] indicated that the chirality was a new relevant operator at the chiral fixed point. Although the chirality orders simultaneously with the spin, characterized by the same chiral correlation-length exponent ν_κ as the spin correlation-length-exponent ν, the chirality carries its own anomalous dimension which is different from the spin anomalous dimension. The chirality has a unique chiral crossover exponent $\phi_\kappa = \beta_\kappa + \gamma_\kappa$, with satisfying the scaling relation $\alpha + 2\beta_\kappa + \gamma_\kappa = 2$. These RG predictions were also supported by the MC results of Ref.[3].

Following the theoretical suggestion of a new chiral universality class, various ex-

periments have also been performed on stacked-triangular antiferromagnets to test the prediction. The critical exponents and the specific-heat amplitude ratio experimentally determined for the XY ($n = 2$) chiral magnets, *e.g.*, $CsMnBr_3$ [14, 15, 17, 18, 16] and $CsNiCl_3$ ($CsMnI_3$) above the critical (multicritical) field [19, 20, 21, 22], are summarized in Table II. All experiments reported that the transition was continuous. Several theoretical values are also shown for comparison, including the $n = 2$ chiral values of Ref.[5]. As can be seen from Table II, these experimental values are consistent with each other, and are in favor of the chiral-universality scenario. We note that the experimental situation in the Heisenberg case is also similar to the XY case: See Refs.[6] and [23] for details.

At this point, I wish to add a few comments. First, even when the chiral fixed point is stable with a finite basin of attraction in the parameter space, it is still entirely possible that some systems possessing the full chiral symmetry exhibit a first-order transition. This occurs only if the initial Hamiltonian characterizing the system happens to lie outside the basin of the chiral fixed point. Here the difference between the two classes of systems, one showing a continuous chiral transition and the other a first-order transition, is not of symmetry origin, but arises simply from the nonuniversal details of the microscopic parameters. Possible examples of chiral systems showing a first-order transition might be certain matrix models with the rigid noncollinear structures. Indeed, the initial RG Hamiltonians describing these matrix models are likely to be located in the runaway regime of the RG flow, outside the stability domain of the chiral fixed point [6]. Conversely, this observation means that, even if one finds a few chiral systems exhibiting a first-order transition, it does not immediately exclude the possibility that still another class of systems with the same chiral symmetry exhibit a continuous transition belonging to chiral universality.

Second, the $\epsilon = 4 - d$ expansion suggests that the critical value of $n_c(d = 3)$ may lie very close to the physical value $n = 2$ or 3. If $n_c(3)$ lies above the physical value only very slightly, there is no stable fixed point in the strict sense (the chiral

fixed point becomes slightly complex-valued). Nevertheless, the associated RG flow behaves for a long period of RG iterations as if there were a stable fixed point. Thus, a "shadow" of the complex-valued chiral fixed point in the real plane attracts the RG flow up to a certain scale, which could be very long, but, eventually, the flow escapes away from such a pseudo-fixed point showing a runaway characteristic of a first-order transition. Physically, this means that the system exhibits a rather well-defined critical behavior governed by the (slightly complex-valued) chiral fixed point for a wide range of temperature, but eventually, the deviation from such a critical behavior sets in for sufficiently close to T_c, and the system exhibits a weak first-order transition. We emphasize here that, as long as $n_c(3)$ lies very close to the physical value, the situation would virtually be indistinguishable for all practical purposes from the case of a truly stable chiral fixed point, in the sense that almost universal novel critical behavior is observed for a variety of chiral systems in a wide temperature range up to very close to T_c [6].

Third, besides the $\epsilon = 4 - d$ and $1/n$ expansions, there is another perturbative RG scheme called the $\epsilon = d - 2$ expansion [13, 24]. This method applied to the $n = 3$ chiral systems yielded a continuous transition characterized by the standard $O(4)$ criticality. Meanwhile, as shown above, such an $O(4)$ behavior has not been observed either numerically nor experimentally in the magnetic ordering of stacked-triangular antiferromagnets. The possible reason of this failure of the $\epsilon = d - 2$ expansion was discussed by several authors [24, 25, 27, 26]. One reason might be that the nonperturbative effects associated with the topological defects, which might play a vitally important role in the phase transition of chiral systems, was completely neglected there [24, 25]. In particular, if the $n = 3$ chiral system in $d = 2$ dimensions exhibits a pure topological phase transition at a finite temperature driven by the dissociation of the Z_2 vortices, as first suggested in Ref.[28] and numerically supported in Refs.[29, 30, 31], the $\epsilon = d - 2$ expansion should completely fail *even at and near two dimensions*. Then, to construct a phase diagram of the chiral system in the entire

$d-n$ plane ($2 \leq d \leq 4$) remains as an open question.

Before concluding this section of stacked-triangular antiferromagnets, I wish to briefly discuss the future problems and challenges, mainly on experimental side.

i) The first obvious direction is to perform high-precision experiments (or numerical simulations) to see whether the chiral transition is either continuous or first order, and to precisely determine the associated critical exponents and amplitude ratio. In particular, if the chiral transition is really of weakly first order, it should eventually show up in high-precision experiments as a finite latent heat or a finite discontinuity in certain physical quantities. In doing this, one has to choose appropriate materials characterized by the full chiral symmetry. Otherwise, a weak but nonzero perturbation of lower symmetry, *e.g.*, the ones arising from the lattice distortion as in the cases of $RbMnBr_3$ and $CsCuCl_3$, would cause a crossover to the non-chiral behavior in a close vicinity of T_c, irrespective of the eventual stability of the chiral fixed point.

ii) The other direction is to further examine the underlying *universality* of the novel chiral critical behavior, *i.e.*, to examine the critical properties from one material to the other. If the novel chiral criticality is to be observed in various chiral magnets in common, it should be a good experimental proof that there indeed exists a chiral fixed point behind the phenomena.

iii) Another interesting point, closely related to the point ii) above, is to examine the critical properties of helical magnets (spiral magnets), which is expected to lie in the same universality class as the stacked-triangular antiferromagnets. So far, the critical properties of helimagnets with nontrivial chiral degrees of freedom have been studied almost exclusively for rare-earth metals, Ho, Dy and Tb. However, the situation for rare-earth helimagnets has remained confused for years now [6]. Apparently, there are serious problems associated with the long-range nature of the RKKY interaction, or with the possible contribution of the second length scale arising from the disorder-containing skin part of the sample. One way to avoid these complications might be to make measurements on *insulating* helimagnets such as VF_2 and

$\beta-MnO_2$, and I urge experimentalists to try such experiments.

iv) Since the chirality is a quantity characteristic of the novel chiral criticality, a direct experimental observation of the chirality would be very useful. Recently, a significant progress in this direction was made by Plakty *et al*, who determined by using elaborate experimental technique the critical behavior of the chirality for the stacked-triangular antiferromagnets $CsMnBr_3$ and $CsNiCl_3$ [32]. The determined chirality exponents are compared with the prediction of the chiral universality quite successfully.

We finally note that, while the chiral degrees of freedom inherent to stacked-triangular antiferromagnets and other chiral magnets give rise to the novel critical behavior, quite possibly lying in a new universality class, the chirality itself behaves as a composite operator of the order parameter of the transition, the spin. In other words, the spin and the chirality are not decoupled here, and there is only one diverging length scale in the transition. Under certain circumstances, however, even this property could change, as we shall see in the next section on the SG ordering.

§4. Ordering in spin glasses

Spin glasses are the type of random magnets in which both ferromagnetic and antiferromagnetic interactions coexist and compete, thereby giving rise to the effects of frustration and quenched randomness. Since the experimental discovery of a sharp transition-like phenomenon in typical SG magnets, particularly in the so-called canonical SG that are dilute metallic alloys such as AuFe and CuMn, their ordering properties have been studied quite extensively both experimentally and theoretically [7]. Nevertheless, the true nature of the SG ordering still remains elusive, and is under hot debate.

Since we now have fairly convincing experimental evidences that typical SG magnets exhibit an equilibrium phase transition at a finite temperature, next obvious question is what is the true nature of the experimentally observed SG transition and

of the low-temperature SG state. We have not yet reached the full understanding of this question. In theoretical studies of the ordering of SG, *e.g.*, of the critical properties of the SG transition or of the issue of whether the SG state exhibits a spontaneous replica-symmetry breaking (RSB), a simple Ising model with short-range interaction, the so-called Edwards-Anderson (EA) model, has widely been used as a "realistic" SG model. One should bear in mind, however, that the magnetic interactions in many SG materials are nearly isotropic, being rather well described by an isotropic Heisenberg model, in the sense that the magnetic anisotropy is considerably weaker than the exchange interaction. In apparent contrast to experiments, numerical simulations have indicated that the standard SG order occurs only at zero temperature in the 3D Heisenberg SG [33, 35, 8, 36, 37]. Although the magnetic anisotropy inherent to real materials is often invoked to explain this apparent discrepancy with experiments, it still remains puzzling that no detectable sign of Heisenberg-to-Ising crossover has been observed in experiments which is usually expected to occur if the observed finite-temperature transition is caused by the weak magnetic anisotropy[7, 33, 34].

In this section, I wish to explain one scenario of the experimental SG ordering presented some time ago by the present author, aimed at solving this apparent puzzle[8]. In this scenario, which may be termed as a spin-chirality decoupling-recoupling scenario, a scalar chirality introduced in §2 plays an important role. This scenario consists of two parts: The fist part is "spin-chirality separation" or "decoupling" for the fully isotropic system, and the second part is the "spin-chirality mixing" or "recoupling" due to random magnetic anisotropy.

Recent numerical simulations indicate that, in a fully isotropic Heisenberg SG in 3D, the chirality is "separated" on long length and time scales from the spin, giving rise to a novel chiral-glass ordered phase in which only the scalar chiralities are ordered in a spatially random manner with keeping the spin paramagnetic [36, 37]. In other words, in a 3D isotropic Heisenberg SG, the chiral-glass order precedes the SG order, $T_{\rm CG} > T_{\rm SG}$, the latter probably taking place only at $T = T_{\rm SG} = 0$. Furthermore, a

recent numerical simulation has revealed that the nature of the chiral-glass transition and of the chiral-glass ordered state might differ in some essential points from that of the standard Ising SG [37]. First, the critical properties of the chiral-glass transition, characterized by the exponents $\beta_{\rm CG} \simeq 1.1,\ \gamma_{\rm CG} \simeq 1.5, \nu_{\rm CG} \simeq 1.2$ and $\eta_{\rm CG} \simeq 0.8$ *etc.*, differ from those of the standard 3D Ising spin glass, characterized by the exponents $\beta \simeq 0.55,\ \gamma \simeq 4.0, \nu \simeq 1.8$ and $\eta \simeq -0.35$, *etc* [44, 45, 46]. Second, the chiral-glass ordered state is likely to exhibit a novel type of replica-symmetry breaking (RSB) *with a one-step-like character*, in contrast to the case of the 3D Ising SG.

Of course, the experimentally observed SG transition is associated with the freezing of the spin itself: Experimentally, the scalar chirality has not been directly measurable in SG so far. Since the chiral-glass order expected in a 3D isotropic Heisenberg SG does not accompany the spin order, it cannot immediately be connected with the experimental SG ordering. At this point, we come to the second part of the scenario, "spin-chirality recoupling" or "mixing". Here, one assumes that the weak random magnetic anisotropy inherent to real SG materials "mixes" or "recouples" the spin and the chirality, and the chiral-glass transition, which is hidden in the chiral sector in the absence of magnetic anisotropy, is now "revealed" in the spin sector. As long as the anisotropy is sufficiently weak, the nature of the SG transition and of the SG ordered state should be governed by that of the chiral-glass transition and of the chiral-glass ordered state in the fully isotropic system. Note that it is *not* governed by the the SG transition and the SG ordered state of the fully isotropic system, since the spin has been separated from the chirality there.

One can see such a spin-chirality mixing from a simple symmetry argument. The random magnetic anisotropy generally reduces the global symmetry of the Hamiltonian from $O(3) = Z_2 \times SO(3)$ to only *chiral* Z_2 associated with the global spin inversion $\vec{S}_i \to -\vec{S}_i$. Note that the spin inversion flips the scalar chirality $\chi_i \to -\chi_i$, since χ_i is cubic in spins. As mentioned, in the fully isotropic case, the spin-chirality separation is expected to occur. There, upon renormalization, the Hamiltonian is

asymptotically decoupled into the Z_2-symmetric chiral part and the $SO(3)$-symmetric part, where the Z_2 chiral part is subject to a finite-temperature chiral-glass transition, while the $SO(3)$ part remains disordered keeping the Heisenberg spin paramagnetic even in the chiral-glass ordered state. Suppose that a small amount of random magnetic anisotropy is added to the system. While the random magnetic anisotropy energetically breaks the $SO(3)$ symmetry, the chiral-glass transition associated with the chiral Z_2 symmetry would persist essentially unchanged, since the random anisotropy preserves the Z_2 chiral symmetry and the Z_2 chiral part has already been separated from the $SO(3)$ part. In contrast to the fully isotropic case, once the chiral-glass transition takes place and the Z_2 symmetry is spontaneously broken, the spin is no longer allowed to remain paramagnetic since the system now does not possess the $SO(3)$ symmetry. Thus, the spin is now "forced to order" in the chiral-glass state via the effective coupling between the spin and the chirality generated by the random magnetic anisotropy.

Based on this physical picture, one can derive various interesting predictions for the real Heisenberg-like SG ordering. For example, the SG transition temperature in the presence of random magnetic anisotropy of magnitude D is expected to behave for small D as

$$T_{\rm SG}(D) \approx T_{\rm CG}(0) \left[1 + c \left(\frac{D}{k_B T_{\rm CG}(0)} \right)^2 + \cdots \right],$$

where $T_{\rm CG}(0)$ is the chiral-glass transition temperature of the fully isotropic system, and c is a numerical constant. Here note that $T_{\rm CG}(0)$ is a quantity of $O(J)$, not of $O(D)$. Hence, in the $D \to 0$ limit, $T_{\rm SG}(D)$ tends to a finite value of $O(J)$ essentially in a regular way. The reason why the $T_{\rm SG}(D)$ in the $D \to 0$ limit tends to $T_{\rm CG}(0)$ associated with the Z_2-symmetry breaking, not to $T_{\rm SG}(0)(< T_{\rm CG}(0))$ associated with the $SO(3)$-symmetry breaking, is because the symmetry spontaneously broken at the SG transition under finite D is the chiral Z_2 symmetry (spin-inversion symmetry), not being the continuous $SO(3)$ symmetry.

The nonlinear susceptibility for sufficiently weak D diverges toward $T = T_{\rm SG}(D)$

as

$$\chi_{nl}(D) \approx D^4 t^{-\gamma_{\rm CG}} + [\text{nondiverging term}],$$

where t is a reduced temperature $t \equiv |(T - T_{\rm SG}(D))/T_{\rm SG}(D)|$. The susceptibility exponent here is nothing but the chiral-glass exponent in the fully isotropic system $\gamma = \gamma_{\rm CG} \simeq 1.5$. The above equation indicates that the standard crossover behavior in the exponent does not occur even for weak D. In contrast, the *amplitude* of the leading singularity depends on D, vanishing in the isotropic limit. In this way, the present chirality scenario gives a natural explanation of the abovementioned puzzles about experiments, *i.e.*, why the expected Heisenberg-to-Ising crossover has not been observed experimentally, or why the SG transition occurs at a finite temperature of order J even in a nearly isotropic Heisenberg-like SG with weak anisotropy.

According to the present chirality scenario, the critical properties of the SG transition of real Heisenberg-like SG should be the same as those of the chiral-glass transition of an isotropic Heisenberg SG, *i.e.*, $\beta \simeq 1.1$, $\gamma \simeq 1.5$, $\nu \simeq 1.2$ and $\eta \simeq 0.8$. These exponent values, though quite different from the values of the 3D Ising SG, happen to be rather close to the experimental values for canonical SG. In Table III, I summarize the SG critical exponents experimentally determined for canonical SG such as AgMn, CuMn *etc*. One sees that the exponents determined by various authors [38, 39, 40, 41, 42, 43] come close to each other, yielding the values $\beta \simeq 1$, $\gamma \simeq 2$, $\nu \simeq 1.3$ and $\eta \simeq 0.5$. In Table III, we also give the corresponding exponent values of the 3D Ising SG determined by recent extensive numerical simulations [44, 45, 46]. Evidently, the experimental values for canonical SG deviate considerably from the 3D Ising values, while they come rather close to the chiral-glass values quoted above, giving some support to the chirality scenario.

It should be noticed that one cannot ascribe the cause of the observed large discrepancy between the experimental and the 3D Ising exponent values to the long-range nature of the RKKY interaction inherent to canonical SG. Although the RKKY interaction is expected to be a relevant perturbation in the spin ordering of 3D Heisenberg

SG [47], a scaling theory suggests that the 3D Heisenberg SG with the RKKY interaction lies just at its lower critical dimension (LCD) [47]. Generally, the system at its LCD is known to exhibit either a zero-temperature transition with an exponentially diverging correlation length, or a finite-temperature Kosterlitz-Thouless-type transition without a finite long-range order but with an exponentially diverging correlation length. In either case, ν should be infinite, which is hardly consistent with the experimental result for canonical SG, $\nu \simeq 1.3$. Furthermore, the non-Ising exponents close to the ones obtained for canonical SG are also observed in an insulating Heisenberg-like SG with short-range interaction, CdCrInS [43]: See Table III. These observations indicate that the cause of the non-Ising exponents experimentally observed in canonical SG cannot simply be ascribed to the RKKY interaction.

Thus, the chirality scenario turns out to naturally explain not only the origin of the experimentally observed SG exponents, but also the reason why the expected crossover has not been observed experimentally, or why the SG transition occurs at a finite temperature of $O(J)$ even in a nearly isotropic Heisenberg-like SG system. In fact, this scenario could also explain some other features where the standard theory met some difficulty, such as the problem of magnetic phase diagram [48]. It also yields some new predictions, *e.g.*, that the SG ordered state of real Heisenberg-like SG should exhibit a peculiar type of one-step-like RSB. Further details of the chirality scenario and its outcome will be given elsewhere [48].

§5. Concluding remark

A brief review has been given on the two topics in the ordering of frustrated magnets where the chirality plays an important role. One is the criticality of stacked-triangular antiferromagnets and the other is the ordering of spin glasses. In both cases, the chirality gives rise to new interesting phenomena not encountered in standard unfrustrated magnets, such as "chiral universality" or "spin-chirality separation". While further works are required to settle the issues discussed in this article, it is

already clear that frustration indeed offers a unique stage where a variety of new interesting ordering phenomena take place.

The author is particularly indebted to Dr. K. Hukushima for useful discussion and collaboration in a work cited in §4.

Note added to §3: Very recently, Pelissetto, Rossi and Vicari performed a RG loop expansion at $d = 3$ up to six loops [A. Pelissetto, P. Rossi and E. Vicari, Phys. Rev. B**63**, R140414 (2001)]. By determining the large-order behavior of the series and making a resummation, these authors found a stable chiral fixed point for both cases of $n = 2$ and 3 characterized by the exponents which were in good agreement with experiments and MC. Note that the result is in sharp contrast to the previous lower-order (three-loops) result where no stable fixed was found.

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Ref.	Kawamura [5]	Bhattacharya *et al* [10]	Mailhot *et al* [11]	Loison *et al* [12]	Heisen-berg	O(4)	MF tri-critical
T_c/J	0.958(4)	0.9576(2)	0.9577(2)	-			
α	0.24(8)	-	-	-	-0.116	-0.22	0.5
β	0.30(2)	0.289(10)	0.285(11)	0.28(2)	0.36	0.39	0.25
γ	1.17(7)	1.176(20)	1.185(3)	1.25(3)	1.387	1.47	1
ν	0.59(2)	0.585(9)	0.586(8)	0.59(1)	0.705	0.74	0.5
A^+/A^-	0.54(20)	-	-	-	1.36	-	0
β_κ	0.55(4)	-	0.50(2)	-	-	-	0.5
γ_κ	0.72(8)	-	0.82(4)	-	-	-	0.5
ν_κ	0.60(3)	-	0.608(12)	-	-	-	0.5

Table I: Critical exponents, specific-heat amplitude ratio and transition temperature as determined by several MC simulations for the stacked-triangular Heisenberg antiferromagnet with the equal strengths of the inter- and intra-plane couplings J. The corresponding values of the standard Heisenberg, $O(4)$ and mean-field (MF) tricritical universality classes are also shown.

	α	β	γ	ν	A^+/A^-
$CsMnBr_3$	0.39(9)[17]	0.22(2)[14]	1.10(5)[15]	0.57(3)[15]	0.19(19)[17]
	0.40(5)[18]	0.25(1)[15]	1.01(8)[14]	0.54(3)[14]	0.32(20)[18]
		0.24(2)[16]			
$CsNiCl_3$	0.37(8)[19]	0.243(5)[21]	-	-	0.30(11)[19]
$(H > H_m)$	0.342(5)[20]				
$CsMnI_3$	0.34(6)[22]	-	-	-	0.31(8)[20]
$(H > H_m)$					
XY	-0.008	0.35	1.316	0.669	0.99
$n = 2$ chiral[5]	0.34(6)	0.253(10)	1.13(5)	0.54(2)	0.36(20)
MF tricritical	0.5	0.25	1.0	0.5	0

Table II: Critical exponents and specific-heat amplitude ratio determined by the experiments on several stacked-triangular XY antiferromagnets. The corresponding values given by several theories are also shown.

type	material	Ref.	β	γ	ν	η
canonical SG	CuMn,AgMn	[39]	1.0(1)	2.2(1)	~ 1.4	~ 0.4
	AgMn	[40]	1.0(1)	2.2(2)	~ 1.4	~ 0.4
	AgMn	[38]	0.9(2)	2.1(1)	~ 1.3	~ 0.4
	CuAlMn	[41]	~ 1.0	~ 1.9	~ 1.3	~ 0.5
	PdMn	[42]	0.90(15)	2.0(2)	~ 1.3	~ 0.4
insulating SG	CdCrInS	[43]	0.75(10)	2.3(4)	~ 1.3	~ 0.2
3D Ising SG	$\pm J$	[44]	~ 0.55	~ 4.0	1.7(3)	-0.35(5)
(MC)	Gaussian	[45]	~ 0.64	~ 4.7	2.0(1.5)	-0.36(6)
	$\pm J$	[46]	~ 0.65	4.1(5)	1.8(2)	-0.26(4)
chiral glass		[37]	1.1(1)	1.5(3)	~ 1.2	~ 0.8

Table III Critical exponents of canonical SG (Heisenberg-like metallic SG) and of Heisenberg-like insulating SG, compared with the corresponding exponents of the 3D short-range Ising SG (EA model) and those of the chiral-glass. Note that the standard scaling relations have been used here to reproduce the full set of exponents from the values reported in the original references.